Telescope and microscope

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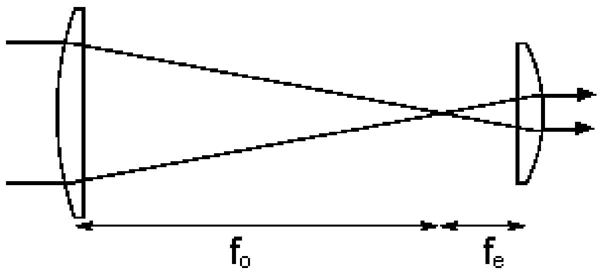
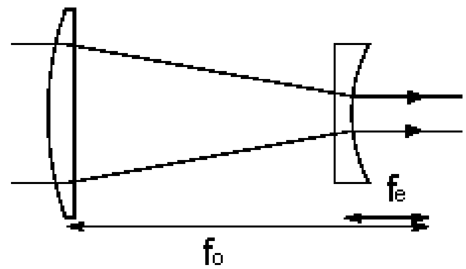
Telescopes and Microscopes

A basic refracting telescope is an optical instrument that has two optical elements, an objective and an eyepiece. We have two thin lenses in air. The objective is a large lens that collects light from a distant object and creates an image in the focal plane, which is a faithful representation of the object. The eyepiece is a sophisticated magnifying glass through which we view this image.



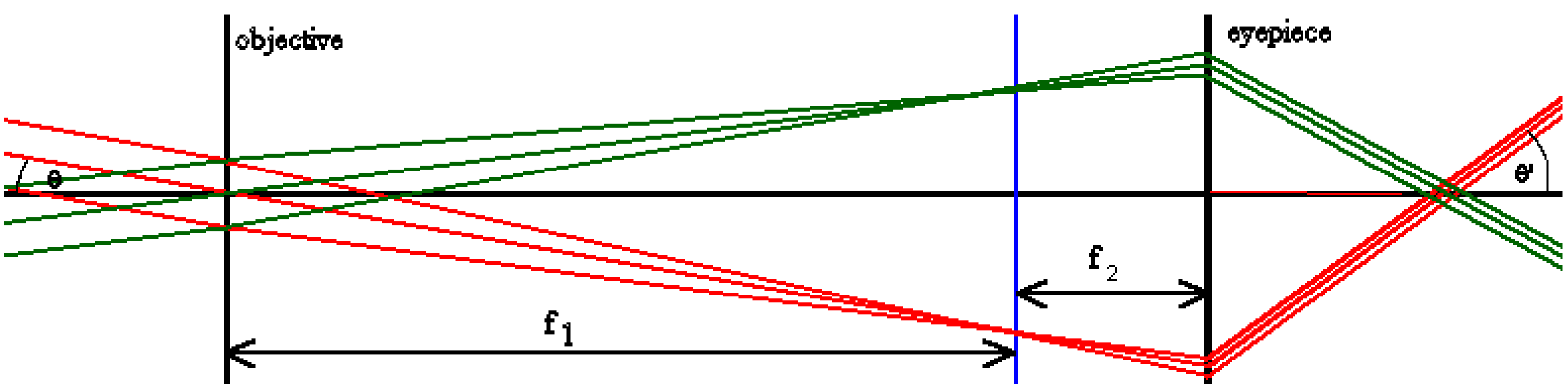
Keplerian telescope



Galilean telescope

A **Keplerian telescope** has a converging lens eyepiece and a **Galilean telescope** has a diverging lens eyepiece. The distance between the image and the eyepiece is the sum of the focal lengths of the two lenses. (Remember that for a diverging lens the focal length is negative.) A telescope by itself is not an image forming system. The eye of the observer or the camera attached to the telescope forms the image.

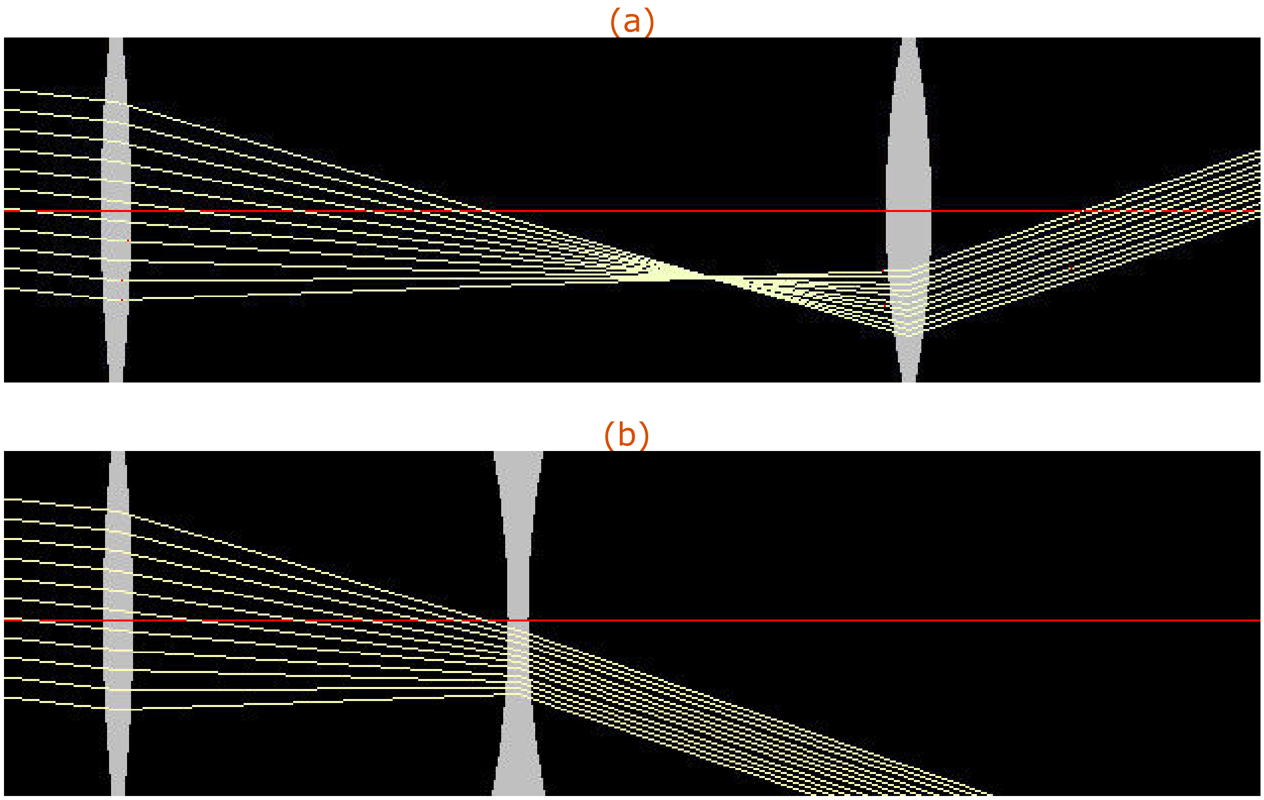
We use a telescope to gather light and to increase the angle that a distant object subtends at the eye. If the eye is relaxed for distant viewing, the telescope simply produces an angular magnification. An incident (approximately) parallel beam from a distant source point, which makes an angle θ with respect to the optical axis, emerges as a parallel beam which makes a larger angle θ' with respect to the axis.



The transformation matrix for the Keplerian telescope is

$$M_{vv} = \begin{pmatrix} 1 & -1/f_2 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ f_1+f_2 & 1 \end{pmatrix} \begin{pmatrix} 1 & -1/f_1 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} -f_1/f_2 & 0 \\ f_1+f_2 & -f_2/f_1 \end{pmatrix}.$$

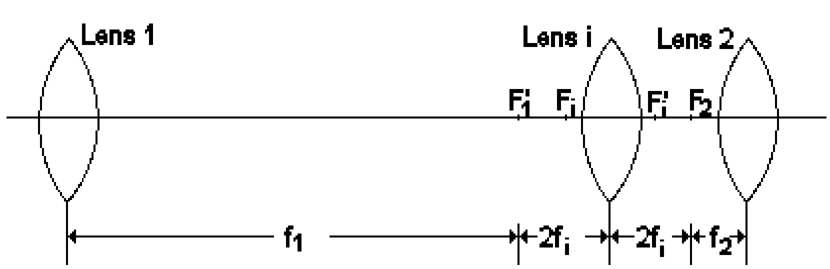
Here f_1 is the focal length of the objective and f_2 is the focal length of the eyepiece. The telescopic system is characterized by $M_{12} = 0$. The angular magnification is $M_{11} = m_\theta = -f_1/f_2$, it is the negative ratio of the focal length of the objective to the focal lengths of the eyepiece.



A Keplerian (a) and a Galilean (b) telescope with the same angular magnification.

The image as viewed through the Keplerian telescope is inverted, and the image formed by the objective lens is in the second focal plane of that lens which is also the first focal plane of the eyepiece lens. The image formed by the eyepiece is at infinity. The telescope is not an image forming system until we add another optical system, such as the lens of an eye or a camera.

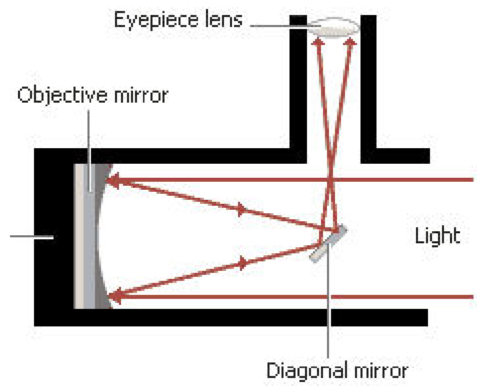
The angular magnification of a Galilean telescope is positive and the image is upright.



For a terrestrial telescope build with only converging lenses, one may insert an erector lens between the objective and eyepiece, such that the image formed by the objective acts as an object for the erector lens which in turn forms an inverted image at the first focal point of the eyepiece lens. The matrix of this system will have M_{12} = 0, and the angular magnification will be positive.

There are two basic types of telescopes:

- A **refractor telescope** that uses a large lens to gather light.
- A **reflector telescope** that uses a large mirror to gather light.



Newtonian Reflecting Telescope

Problem:

A Galilean telescope has an objective lens with f_1 = 20 cm and the eyepiece lens with f_2 = -5 cm. The lenses are separated by 15 cm. Calculate the matrix for this system and find m_θ.

- Solution:
  The lenses are separated by a distance $f_1 + f_2$. Therefore

$$M_{vv} = \begin{pmatrix} -f_1/f_2 & 0 \\ f_1 + f_2 & -f_2/f_1 \end{pmatrix} = \begin{pmatrix} 4 & 0 \\ 15 & 1/4 \end{pmatrix}$$

$M_{11} = m_\theta = +4$ is the angular magnification.

Gathering as much light as possible is the principal function of astronomical telescopes. The larger the area $\pi(D/2)^2$ of the objective the more light the telescope can collect. To compare the light gathering power (LGP) of two telescopes we compare the areas of their objectives. For the eye the objective is just the pupil, with a diameter of only ~0.8 cm.

The resolving power or resolution of a telescope is the smallest angular separation between two objects that can be seen. The smaller the resolving power of the telescope the better is the telescope. If two objects in the sky have a smaller angular separation than the resolving power of the telescope, the two objects will appear as one. However, if their angular separation is larger than the resolving power, then they will be resolved and appear as two distinct objects.

The resolution depends on the wavelength of the light entering the objective as well as on the diameter of the objective. The bigger the diameter of the objective the smaller the resolving power. For a telescope the theoretical intrinsic minimum angular separation is given by

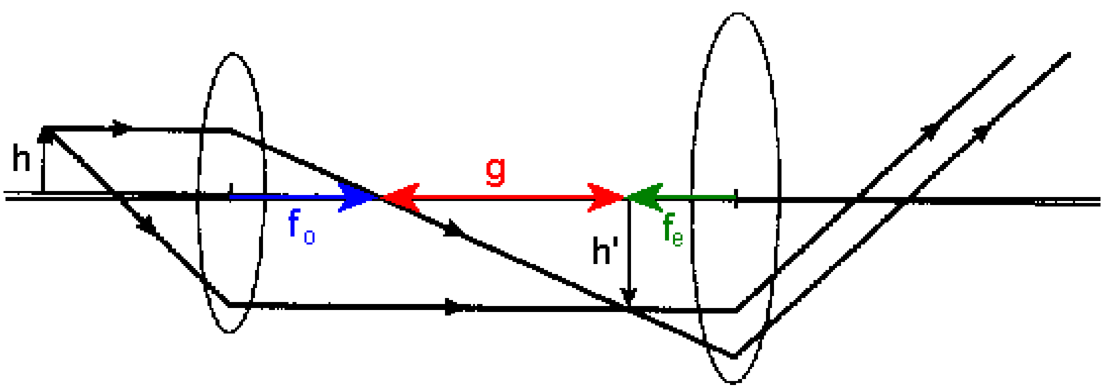
$$\theta_{min} = 1.22\ \lambda/D$$

where λ is the wavelength of the light and D the diameter of the objective. For the human eye and visible light D = 0.8 cm and λ = 500 nm, therefore $\theta_{min} = 7.62*10^{-5}$ rad = $4.37*10^{-3}$ degree. Earth-bound telescopes rarely reach their theoretical resolution limit because of the blurring of the images due to atmospheric turbulence.

The magnifying power MP of a telescope is the (apparent) increase in the size of an object relative to its size when viewed with the unaided eye. MP is equal to the magnitude of the angular magnification $m_\theta = -f_1/f_2$. The ratio of the diameter of the entrance pupil to the exit pupil is also equal to MP. The magnifying power should be chosen so that the diameter of the exit pupil is approximately equal to the diameter of the pupil of the eye.

Microscopes

A compound microscope uses a simple combination of two converging lenses to produce a very effective magnifier. A sketch is shown below. The lens closest to the object is known as the objective, and the second lens is the eyepiece. The object is placed between f_o and $2f_o$. An intermediate image is formed by the objective lens near the object focal plane of the eyepiece. The **tube length** g is the distance between the secondary focal point of the objective and the primary focal point of the eyepiece. The intermediate image serves as the object for the eyepiece.



To have a tube length g an object distance x_o satisfying $1/x_o = 1/f_o - 1/(g + f_o)$ is needed. If the intermediate image is in the object focal plane of the eyepiece then the linear magnification of the objective is $-g/f_0$ and and the angular magnification of the eyepiece is d_v/f_e. The total magnifying power of the microscope is given by $MP = -g\ d_v/(f_of_e) = -(g/f_o)MP_{eyepiece}$. Here d_v = 25 cm, the near point of the human eye. The focal lengths of both lenses should be extremely short to generate maximum angular magnification.

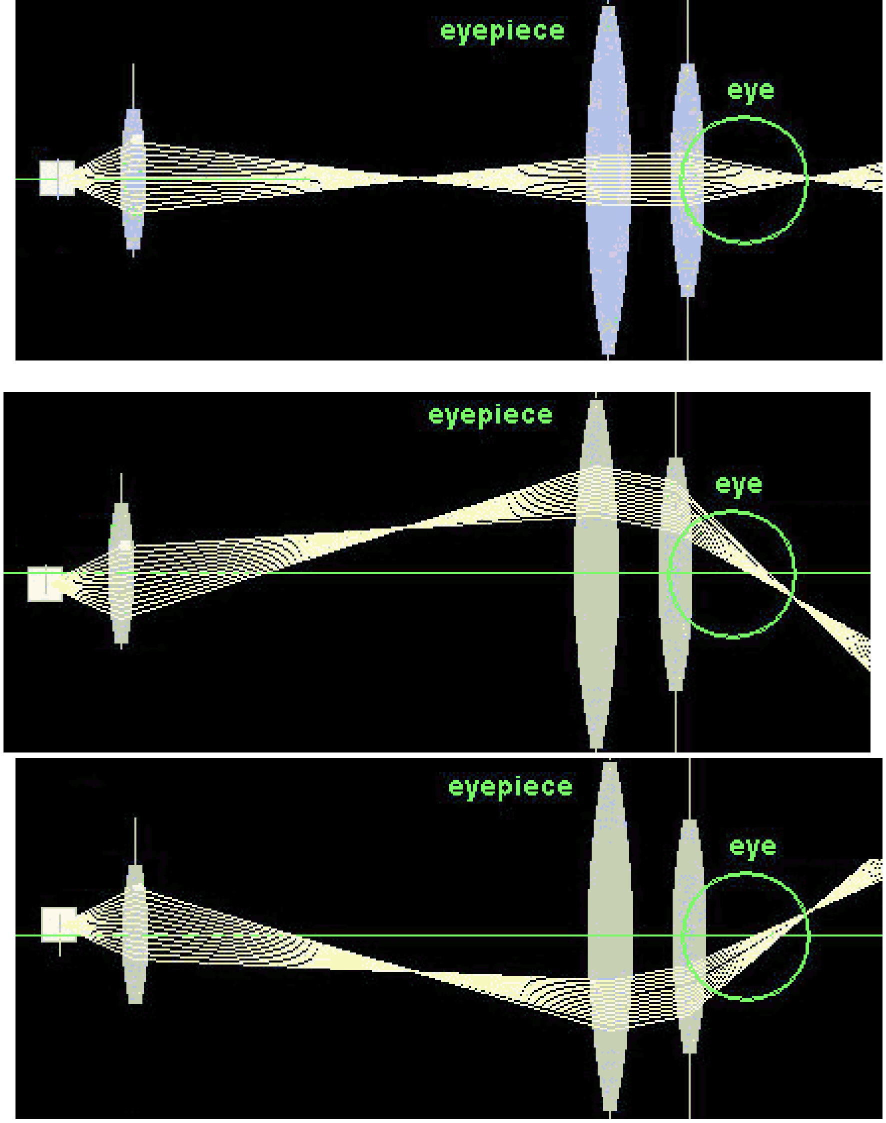


Illustration of the angular magnification of a microscope

Optical instruments

Making things look bigger

When you use an optical instrument, whether it be something very simple like a magnifying glass, or more complicated like a telescope or microscope, you're usually trying to make things look bigger so you can more easily see fine details. One thing to remember about this is that if you want to make things look bigger, you're always going to use converging mirrors or lenses. Diverging mirrors or lenses always give smaller images.

When using a converging lens, it's helpful to remember these rules of thumb. If the object is very far away, the image will be tiny and very close to the focal point. As the object moves towards the lens, the image moves out from the focal point, growing as it does so. The object and image are exactly the same size when the object is at 2F, twice the focal distance from the lens. Moving the object from 2F towards F, the image keeps moving out away from the lens, and growing, until it goes to infinity when the object is at F, the focal point. Moving the object still closer to the lens, the image steadily comes in towards the lens from minus infinity, and gets smaller the closer the object is to the lens.

Note that similar rules of thumb apply for a converging mirror, too.

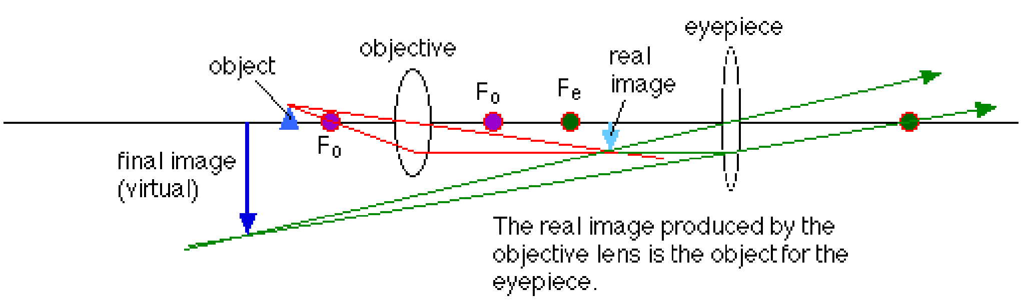
Multiple lenses

Many useful devices, such as microscopes and telescopes, use more than one lens to form images. To analyze any system with more than one lens, work in steps. Each lens takes an object and creates an image. The original object is the object for the first lens, and that creates an image. That image is the object for the second lens, and so on. We won't use more than two lenses, and we can do a couple of examples to see how you analyze problems like this.

A microscope

A basic microscope is made up of two converging lenses. One reason for using two lenses rather than just one is that it's easier to get higher magnification. If you want an overall magnification of 35, for instance, you can use one lens to magnify by a factor of 5, and the second by a factor of 7. This is generally easier to do than to get magnification by a factor of 35 out of a single lens.

A microscope arrangement is shown below, along with the ray diagram showing how the first lens creates a real image. This image is the object for the second lens, and the image created by the second lens is the one you'd see when you looked through the microscope.



Note that the final image is virtual, and is inverted compared to the original object. This is true for many types of microscopes and telescopes, that the image produced is inverted compared to the object.

An example using the microscope

Let's use the ray diagram for the microscope and work out a numerical example. The parameters we need to specify are:

$d_o = 7.0$ mm $\quad$ $h_o = 1.0$ mm

$f_o = 4.5$ mm $\quad$ $f_e = 10.0$ mm

distance between the two lenses = 20 mm

To work out the image distance for the image formed by the objective lens, use the lens equation, rearranged to:

$d_i = d_o f / (d_o - f) = (7)(4.5) / (2.5) = 12.6$ mm

The magnification of the image in the objective lens is:

$m_o = - d_i / d_o = -12.6 / 7.0 = -1.8$

So the height of the image is -1.8 x 1.0 = -1.8 mm.

This image is the object for the second lens, and the object distance has to be calculated:

$d_o = 20 - 12.6 = 7.4$ mm

The image, virtual in this case, is located at a distance of:

$d_i = d_o f / (d_o - f) = (7.4)(10) / (-2.6) = -28.5$ mm

The magnification for the eyepiece is:

$m_e = - d_i / d_o = 28.5 / 7.4 = 3.85$

So the height of the final image is -1.8 mm x 3.85 = -6.9 mm.

The overall magnification of the two lens system is:

$m = m_o \times m_e = -1.8 \times 3.85 = -6.9$

This is equal to the final height divided by the height of the object, as it should be. Note that, applying the sign conventions, the final image is virtual, and inverted compared to the object. This is consistent with the ray diagram.

Telescopes

A telescope needs at least two lenses. This is because you use a telescope to look at an object very far away, so the first lens creates a small image close to its focal point. The telescope is designed so the real, inverted image created by the first lens is just a little closer to the second lens than its focal length. As with the magnifying glass, this gives a magnified virtual image. This final image is also inverted compared to the original object. With astronomical telescopes, this doesn't really matter, but if you're looking at something on the Earth you generally want an upright image. This can be obtained with a third lens.

Note that the overall effect of the telescope is to magnify, which means the absolute value of the magnification must be larger than 1. The first lens (the objective) has a magnification smaller than one, so the second lens (the eyepiece) must magnify by a larger factor than the first lens reduces by. To a good approximation, the overall magnification is equal to the ratio of the focal lengths. With o standing for objective and e for eyepiece, the magnification is given by:

$m = - f_o / f_e$, with the minus sign meaning that the image is inverted.

Resolving power

The resolving power of an optical instrument, such as your eye, or a telescope, is its ability to separate far-away objects that are close together into individual images, as opposed to a single merged image. If you look at two stars in the sky, for example, you can tell they are two stars if they're separated by a large enough angle. Some stars, however, are so close together that they look like one star. You can only see that they are two stars by looking at them through a telescope. So, why does the telescope resolve the stars into separate objects while your eye can not? It's all because of diffraction.

If you look at a far-away object, the image of the object will form a diffraction pattern on your retina. For two far-away objects separated by a small angle, the diffraction patterns will overlap. You are able to resolve the two objects as long as the central peaks in the two diffraction patterns don't overlap. The limit is when one central peak falls at the position of the first dark fringe for the second diffraction pattern. This is known as the Rayleigh criterion. Once the two central peaks start to overlap, in other words, the two objects look like one.

The size of the central peak in the diffraction pattern depends on the size of the aperture (the opening you look through). For your eye, this is your pupil. A telescope, or even a camera, has a much larger aperture, and therefore more resolving power. The minimum angular separation is given by:

$\theta_{min} = 1.22 \lambda / D$ where D is the size of the aperture.

The factor of 1.22 applies to circular apertures like your pupil, a telescope, or a camera lens.

The closer you are to two objects, the greater the angular separation between them. Up close, then, two objects are easily resolved. As you get further from the objects, however, they will eventually merge to become one.